This exam will be conducted according to the Georgia Tech Honor Code. I pledge to neither give nor receive unauthorized assistance on this exam and to abide by all provisions of the Honor Code.

## Signed (With your Full Legal Name):

Instructions: Instructions: This is a closed book, closed note exam. Calculators are not permitted. If you have a question, raise your hand and I will come to you. Please work the exam in pencil and do not separate the pages of the exam. For maximum credit, show your work.

Good Luck!

Your Name (please print): $\qquad$

For maximum (and partial) credit, show your work.

Good Luck!


## Combinational Building Blocks



Problem 1 (5 parts, 20 points)
Part A (8 points) Convert the following notations using the minimum required digits.

| Binary Notation | Numbering System | Decimal |
| :---: | :---: | :---: |
| $\mathbf{1 1 0 1 0 1 0}$ | Signed-magnitude | $\mathbf{- 4 2}$ |
| 10001 | 1's complement | $\mathbf{- 1 4}$ |
| $\mathbf{0 1 1 0 1}$ | 2's complement | 13 |
| 101110 | 2's complement | $\mathbf{- 1 8}$ |

Part B (6 points) For the binary representations below, determine the range (minimum to maximum values).
Express all answers in decimal notation.

| Representation | Minimum Value | Maximum Value |
| :---: | :---: | :---: |
| Unsigned integer (6 bits) | $\mathbf{0}$ | $\mathbf{6 3}$ |
| 1's complement (5 bits) | $\mathbf{- 1 5}$ | $\mathbf{1 5}$ |
| 2's complement (3 bits) | $\mathbf{- 4}$ | $\mathbf{3}$ |

Part C (2 points) In combinational logic circuits, the outputs are determined by both inputs and the current state (memory) of the system.
a) True
b) False

Part D (2 points) In pass (or transmission) gates, current follows a different path based on whether the input is high or low.
a) True
b) False

Part E (2 points) Which one of the following building blocks cannot be used to cascade (or tie multiple building blocks together to build multi-bit versions of the same function)?
a) Decoder
b) Priority Encoder
c) Demultiplexer
d) Half-adder

## Problem 2 (2 parts, 20 points)

Do NOT change the order of the operands, show all work, and CIRCLE the final answer.
PART A (10 points) Show how $31+9$ is done in 6 -bit 2 's complement. Is your answer correct? EXPLAIN why or why not? Did an overflow occur?
31
+9

40 $\quad$| 11111 |
| :---: |
| 011111 |
| +001001 |
| 101000 |$\quad$ Overflow? YES

The answer is incorrect because an overflow occurred. The sign bit was pushed into the 6th bit so that the answer is saved as -24 instead of +40 .

PART B (10 points) Show how $15-28$ is done in 6-bit 2's complement. Is your answer correct? EXPLAIN why or why not? Did an overflow occur?

| 11 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 15 | => | 001111 | Overflow? | NO |
| +-28 |  | +100100 |  |  |
| -13 |  | 110011 |  |  |

The answer is correct because the final result is a negative number (no overflow has occurred) and is equal to -13 (110011 -> -001101->-13)

Problem 3 (4 parts, 40 points)
Solve the following questions given the truth table below.

| $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{X}$ | $\mathbf{Y}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 |

PART A (10 points) Design output $\mathbf{X}$ using a single 3-to- 8 decoder (without an enable) and the minimum number of gates. You may use any logic gate with no limit on the number of gate inputs. Do not simplify.


PART B (10 points) How many transistors would you need to implement the circuit above? Assume that the decoder is implemented using canonical sum-of-products equations (no minimization), complemented inputs are provided inside the decoder, and logic gates have no limit on the number of gate inputs.

Implementing decoders using canonical SOP equations requires eight 3-input AND gates. If you assumed that complemented inputs are provided, you only need 6 transistors per 3-input AND gate. I also accepted you using 8 transistors per 3-input AND gate ( 6 transistors for a NAND and 2 transistors for a NOT gate).
The total number of transistors needs to include 10 transistors to account for the 4-input OR gate.
\# transistors, decoder $=6 \mathrm{~T} \times 8 \mathrm{AND}_{3-\mathrm{inp}}=\mathbf{4 8}$ transistors or $8 \mathrm{~T} \times 8 \mathrm{AND}_{3 \text {-input }}=\mathbf{6 4}$ transistors

$$
\# \text { transistors, total }=48 \text { transistors }+10 \mathrm{~T} \times 1 \mathrm{OR}_{4-\mathrm{inp}}=58 \text { transistors or } \mathbf{7 4} \text { transistors }
$$

Problem 3 Continued (4 parts, 40 points)
Decoder / Multiplexer Design
Solve the following questions given the truth table below.

| $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{C}$ | $\mathbf{X}$ | $\mathbf{Y}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 |

PART C (10 points) Design output $\mathbf{Y}$ using a single 8-to-1 multiplexer (without an enable) and the minimum number of gates. You may use any logic gate with no limit on the number of gate inputs. Do not simplify.


PART D (10 points) How many transistors would you need to implement the circuit above? Assume that the multiplexer is implemented using pass gates and complemented inputs are provided.

Multiplexers are built using banks of pass gates to downselect to a single output signal. The first bank will need 8 pass gates, the second bank will need 4 pass gates, and the third and final bank needs 2 pass gates.
The total number of transistors is the same as the number needed for the multiplexer design.

$$
\text { \# transistors, decoder }=2 \mathrm{~T} \times 14 \text { Pass }=28 \text { transistors }
$$

\# transistors, total $=28$ transistors

Problem 4 (3 parts, 20 points)
PART A (4 points) Consider a priority encoder with the following behavior:


| $\mathbf{I}_{\mathbf{3}}$ | $\mathbf{I}_{\mathbf{2}}$ | $\mathbf{I}_{\mathbf{1}}$ | $\mathbf{I}_{\mathbf{0}}$ | $\mathbf{V}$ | $\mathbf{O}_{\mathbf{1}}$ | $\mathbf{O}_{\mathbf{0}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | X | X |
| X | X | X | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 1 |
| X | 1 | X | 0 | 1 | 1 | 0 |
| 1 | 0 | X | 0 | 1 | 1 | 1 |

What input priority has been assigned to the four inputs $\left(\mathrm{I}_{3}, \mathrm{I}_{2}, \mathrm{I}_{1}\right.$, and $\left.\mathrm{I}_{0}\right)$ ?

$$
\begin{array}{lllllll}
\mathbf{I}_{0} & > & \mathbf{I}_{\mathbf{2}} & > & \mathbf{I}_{3} & > & \mathbf{I}_{1}
\end{array}
$$

PART B (6 points) Build an 8-bit magnitude comparator using the two 4-bit building blocks below. You should label inputs ( $\mathrm{A}_{0-7}, \mathrm{~B}_{0-7}, \mathrm{AEQB}_{\text {IN }}, \mathrm{ALTB}_{\text {IN }}$, and $\mathrm{AGTB}_{\text {IN }}$ ) and outputs (AEQBout, ALTBout, and AGTB $_{\text {out }}$ ) as well as show connections between the chips.


PART C (10 points) Use at least two types of building blocks (e.g., decoders, encoders, multiplexers, adders, comparators) and logic gates to design the 1-bit Mini Arithmetic Logic Unit (ALU) described below.


