Problem 1 (3 parts, 24 points)
Part A ( 6 points) Define a 1 to 2 decoder by completing the behavior table.

| IN | EN | O0 | O1 |
| :---: | :---: | :---: | :---: |
| $X$ | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 |



Part B (8 points) Implement a 1 to 2 decoder using basic gates. Assume only true (noncomplemented) inputs are available. Label all inputs and outputs.


Part C (10 points) Using only the three 1 to 2 decoders shown below, implement a 2 to 4 decoder with an enable. Label the decoder inputs $\left(\mathrm{IN}_{1}, \mathrm{IN}_{0}, \mathrm{EN}\right)$ and outputs (O0, O1, O2, O3).


Problem 2 (2 parts, 18 points)
"Get your priorities right!"
Consider a priority encoder with the following behavior:

| $\mathrm{In}_{3}$ | $\mathrm{In}_{2}$ | $\mathrm{In}_{1}$ | $\mathrm{In}_{0}$ | Valid | $\mathrm{O}_{1}$ | $\mathrm{O}_{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | X | X |
| X | X | X | 1 | 1 | 0 | 0 |
| X | X | 1 | 0 | 1 | 0 | 1 |
| X | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 |

Part A (8 points) List the inputs $\left(\operatorname{In}_{0}, \operatorname{In}_{1}, \operatorname{In}_{2}\right.$, and $\left.\mathrm{In}_{3}\right)$ in increasing priority.

$$
\frac{I n_{3}}{\text { lowest priority }}<\frac{I n_{2}}{3^{\text {rd }} \text { highest priority }}<\frac{I n_{1}}{2^{\text {nd }} \text { highest priority }}<\frac{I n_{0}}{\frac{\text { highest priority }}{}}
$$

Part B ( 10 points) Express the behavior of $\mathrm{O}_{0}$ in the map below. Derive a simplified sum of products expression using a Karnaugh Map. Circle and list the prime implicants, indicating which are essential. Then write the simplified SOP expression.

$\mathrm{O}_{0}=$
$\overline{\operatorname{InO}} \cdot(\operatorname{In} 1+\overline{\operatorname{In} 2})$

Problem 3 (4 parts, 40 points)
Number Systems \& Arithmetic
Part A (10 points) Convert the following notations:

| binary notation | decimal notation |
| :---: | :---: |
| 11011011. | 219 |
| 1011100.1101 | 92.8125 |
| 11011.101 | 27.625 |
| binary notation | hexadecimal notation |
| 100100101.110111 | $125 . D C$ |
| 110010110100.001010110001 | CB4.2B1 |

Part B (12 points) For the 22 bit representations below, determine the most positive value and the step size (difference between sequential values). All answers should be expressed in decimal notation. Fractions (e.g., 3/16ths) may be used. Signed representations are two's complement.

| representation | most positive value | step size |
| :---: | :---: | :---: |
| unsigned integer <br> (22 bits) . (0 bits) | 4 M | 1 |
| signed fixed-point <br> (18 bits) . (4 bits) | 128 K | $1 / 16$ |
| signed fixed-point <br> (14 bits) . ( bits) | 8 K | $1 / 256$ |
| signed fixed-point <br> $(11$ bits) . (11 bits) | 1 K | $1 / 2 \mathrm{~K}$ |

Part C (6 points) A 16 bit floating point representation has a 10 bit mantissa field, a 5 bit exponent field, and one sign bit.

$$
\begin{array}{lc}
\text { What is the largest value that can be represented (closest to infinity)? } & 2^{15} \\
\text { What is the smallest value that can be represented (closest to zero)? } & 2^{-16} \\
\text { How many decimal significant figures are supported? } & 3
\end{array}
$$

Part D (12 points) For each problem below, compute the operations using the rules of arithmetic, and indicate whether an overflow occurs assuming all numbers are expressed using a five bit unsigned fixed-point and five bit two's complement fixed-point representations.

|  | 10.11 | 111.10 | 100.01 | 1.11 |
| :---: | :---: | :---: | :---: | :---: |
|  | +11.01 | $\begin{array}{r}11.01 \\ +\quad 1.01 \\ \hline\end{array}$ | $\underline{-10.11}$ | -10.00 |
| result | 110.00 | 000.11 | 001.10 | 111.11 |
| unsigned error? |  |  |  |  |

Problem 4 (3 parts, 18 points)
"Does this register?"
Part A (6 points) Express the behavior of the circuit below. Use standard symbols ( $0,1, \mathrm{X}, \mathrm{Z}_{0}$, $\mathrm{Q}_{0}$, etc.). Then name the circuit.


| $A$ | $B$ | $X$ | $Y$ |
| :---: | :---: | :---: | :---: |
| 0 | 0 | $Q_{0}$ | $\bar{Q}_{0}$ |
| 1 | 0 | $Q_{0}$ | $\bar{Q}_{0}$ |
| 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

This circuit is a
Part B (6 points) Implement a register below using only latches, pass gates, and inverters (all in icon form). Complete the behavior table at right. Recall that the CLK signal indicates a full $\Phi_{1} \Phi_{2}$ cycle; so the output should be the value at the end of a cycle (for the given inputs).

| In | WE | Clk | Out | $\overline{\text { Out }}$ |
| :---: | :---: | :---: | :---: | :---: |
| $A$ | 0 | $\uparrow \downarrow$ | $Q_{0}$ | $\bar{Q}_{0}$ |
| $A$ | 1 | $\uparrow \downarrow$ | $A$ | $\bar{A}$ |



Part C (6 points) Assume the following signals are applied to your register. Draw the output signal Out. Draw a vertical line where $\mathbf{I n}$ is sampled. Draw crosshatch where Out is unknown.


